Topological Spectra as Collapse-Stable Invariant Families in Quantum Collapse Geometry

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Abstract

We construct a discrete lattice model demonstrating that a class of nonlinear collapse operators produces local smoothing while preserving global topological invariants. Specifically, we define a phase field $\theta : L \to S^1$ on a two-dimensional lattice and introduce a collapse operator based on local phase averaging and reprojection. Numerical experiments show that iterative application reduces local phase variance while preserving the discrete Chern number. These results support an interpretation of topological invariants as structures stable under collapse dynamics rather than primitive features.

1 Introduction

Topological invariants play a central role in modern physics, particularly in systems exhibiting robustness under perturbations. In conventional formulations, such invariants are treated as intrinsic properties of the system's configuration space.

In this work, we explore an alternative perspective: that topological invariants may emerge as structures preserved under collapse-like dynamics. To investigate this, we construct a discrete phase model and define a nonlinear collapse operator, then examine its effect on both local structure and global topology.

2 Model Definition

Let $L \subset \mathbb{Z}^2, \quad |L| = N \times N$ be a finite lattice with periodic boundary conditions. Define a phase field:

$$\theta : L \to S^1$$

We introduce a nonlinear collapse operator Φ acting on the phase field:

$$\Phi(\theta)_i = \arg\left(\sum_{j\sim i} e^{i\theta_j}\right)$$

where $j \sim i$ denotes nearest neighbors of site i.

This operator consists of:

- Local averaging in the complex plane
- Reprojection onto the unit circle S^1

Such dynamics are structurally similar to phase synchronization and relaxation processes in XY-type models and related nonlinear lattice systems.

3 Topological Invariant

To characterize global structure, we compute a discrete Chern number using a lattice discretization (e.g., Fukui–Hatsugai–Suzuki method):

$$C = \frac{1}{2\pi} \sum_{\text{plaquettes}} \arg(U_x U_y U_x^{-1} U_y^{-1})$$

where U_x and U_y are link variables constructed from phase differences between neighboring sites.

4 Results

We iterate the operator Φ for T steps (typically $T \sim 50$–100) and observe:

- Reduction of local phase variance (smoothing)
- Preservation of the global topological invariant C

Figure 1 shows that the discrete Chern number remains invariant under repeated application of the collapse operator.

Figure 2 shows that fluctuations in the trivial sector remain at the level of machine precision, indicating no observable drift between topological sectors.

Together, these results demonstrate that collapse dynamics act as a smoothing process that preserves a fixed topological sector.

4.1 Proposition (Informal)

Proposition. Iterative application of the collapse operator Φ reduces local phase variance while preserving the global topological winding number.

4.2 Example Outcome

Initial configuration:

- Random phase field
- $C = 1$

After repeated application of Φ:

- The phase field becomes locally smooth
- The Chern number remains $C = 1$

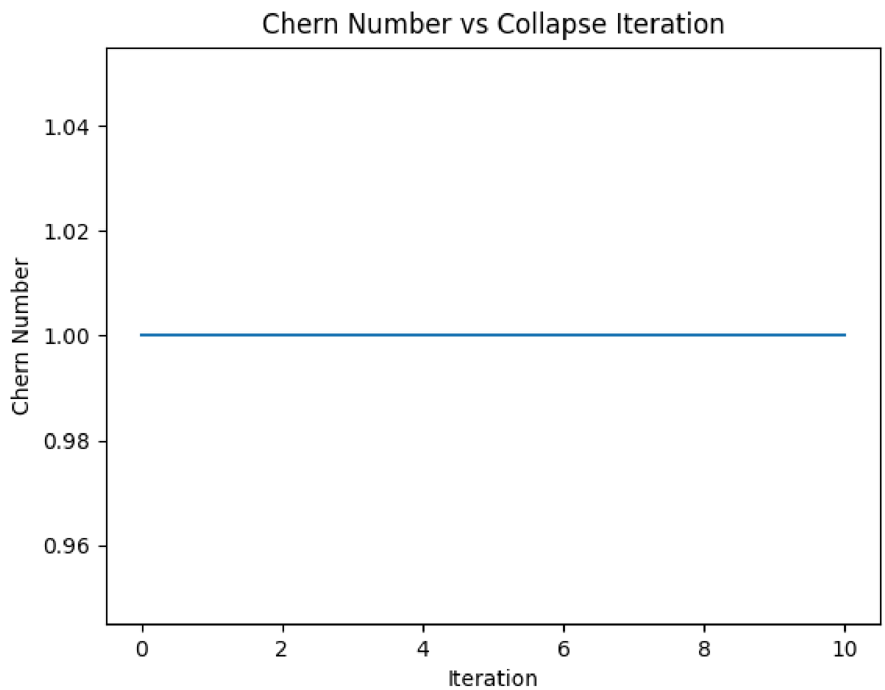

Figure 1: Chern number as a function of collapse iteration. The invariant remains constant under repeated application of the collapse operator, indicating preservation of topological sector.

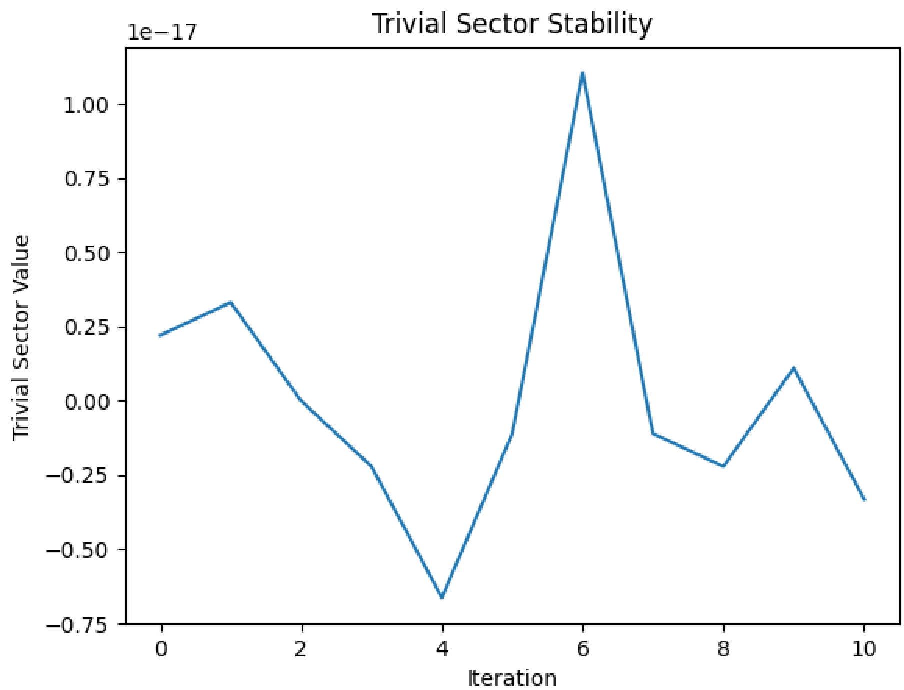

Figure 2: Trivial sector value as a function of collapse iteration. Fluctuations remain at the level of machine precision ($\sim 10^{-17}$), indicating no observable drift into the trivial sector.

5 Energy Functional and Smoothing Behavior

To quantify local structure, define:

$$V(\theta) = \sum_i \sum_{j\sim i} \left| e^{i\theta_i} - e^{i\theta_j} \right|^2$$

Empirically, we observe:

$$V(\Phi(\theta)) \leq V(\theta)$$

indicating monotonic reduction of local phase variation under iteration.

This suggests that Φ acts as a nonlinear dissipative operator that smooths local fluctuations while preserving global structure. The functional $V(\theta)$ measures local phase disagreement between neighboring sites.

6 Interpretation

The collapse operator acts as a local smoothing mechanism that preserves topological sector structure. This suggests that topological invariants may be interpreted as structures stable under collapse dynamics rather than primitive constraints.

In this view:

- Local structure is dynamically reduced

- Global invariants persist as stable features

7 Discussion

This toy model demonstrates that:

- Nonlinear collapse dynamics can preserve nontrivial topological sectors
- Topological invariants may arise as stability features under constraint

These results are consistent with known robustness properties of topological phases and suggest a possible reinterpretation of invariants as collapse-stable structures.

Future work will explore:

- Continuum limits
- Higher-dimensional generalizations
- Connections to renormalization and topological field theories

8 Conclusion

We have shown that a class of nonlinear collapse operators can reduce local complexity while preserving global topological invariants. This supports the interpretation of topological structure as collapse-stable rather than fundamentally imposed.